Exam Information

Subject: Statistics 03

Level: 2nd Year Economy

Academic Year: 2024/2025

University: University of Khenchela

Full Name:………………… **MODEL ANSWER**……$\frac{20}{20}$…………………..

Exercise 01 (07p):

Let X be a random variable representing the number of car accidents occurring in a year. If the mean number of traffic accidents per year is 11, find the following:

1. The probability distribution. **2.** The shape of the probability distribution. **3.** E(x), V(x), δ(x).

.**1.** the random variable X follows a **Poisson distribution** (1p) **because**

. **the number of accidents is a count over a fixed time period** (1p).

.$X \sim Poi(\lambda)$ … (0.5p) …. $X \sim Poi(11)$ … … …. . $since\ \lambda = 11$………(0.5p)…

$P(X = x) = \frac{\lambda^x}{x!}\ e^{-\lambda}$(0.5p)..$P(X = x) = \frac{11^x}{x!}\ e^{-11}$…x=0, 1, 2, 3 (0.5p)..

.**2**. the shape of Poisson distribution depends on the mean $\lambda = 11 > 10$(0.5p)

.is **moderately right-skewed** (0.5p) because when the mean increases the.

. shape becomes more symmetric **but always positive skewed**(0.5p).

3.E(X)=λ,E(X)= **11**, 0.5pV(X)=E(X)= λ=11 0.5p $\delta(x) = \sqrt{V(X)} = \mathbf{3.316}$ 0.5p.

Exercise 02 (06p):

Let X be a random variable representing the annual growth rate of a country's GDP, which follows a normal distribution N(μ,σ), which means that X~N(μ,σ), where μ is the mean growth rate and σ is the standard deviation. Calculate the following probabilities:

1. $P(\mu - \sigma \le X \le \mu + \sigma)$ **2.** $P(\mu - 2\sigma \le X \le \mu + 2\sigma)$ **3.** $P(\mu - 3\sigma \le X \le \mu + 3\sigma)$

…**1**. $P(\mu - \sigma \le X \le \mu + \sigma) = p\left(\frac{(\cancel{\mu}-\sigma)-\cancel{\mu}}{\sigma} \le Z \le \frac{(\cancel{\mu}-\sigma)+\cancel{\mu}}{\sigma}\right)$……(1p)…

$P\left(\frac{-\cancel{\sigma}}{\cancel{\sigma}} \le Z \le \frac{\cancel{\sigma}}{\cancel{\sigma}}\right) = P(-1 \le Z \le 1) = P(Z \le 1) - [1 - P(Z \le 1)]$. (0.5p)

…= 0.84134 - $[1 - 0.84134] = 0.68268 = \mathbf{68.26}\%$…….(0.5p)…………

…**2**. $P(\mu - 2\sigma \le X \le \mu + 2\sigma) = p\left(\frac{(\cancel{\mu}-2\sigma)-\cancel{\mu}}{\sigma} \le Z \le \frac{(\cancel{\mu}-2\sigma)+\cancel{\mu}}{\sigma}\right)$…(1p)….

$P\left(\frac{-2\cancel{\sigma}}{\cancel{\sigma}} \le Z \le \frac{2\cancel{\sigma}}{\cancel{\sigma}}\right) = P(-2 \le Z \le 2) = P(Z \le 2) - [1 - P(Z \le 2)]$ (0.5p)

…0.9772 - $[1 - 0.0228] = 0.9544 = \mathbf{95.44}\%$… (0.5p)…..

3. $P(\mu - 3\sigma \le X \le \mu + 3\sigma) = p\left(\frac{(\cancel{\mu}-2\sigma)-\cancel{\mu}}{\sigma} \le Z \le \frac{(\cancel{\mu}-3\sigma)+\cancel{\mu}}{\sigma}\right)$. (1p)

$P\left(\frac{-3\cancel{\sigma}}{\cancel{\sigma}} \le Z \le \frac{3\cancel{\sigma}}{\cancel{\sigma}}\right) = P(-3 \le Z \le 3) = P(Z \le 3) - [1 - P(Z \le 3)]$. (0.5p)

0.9987 - $[1 - 0.9978] = 0.9974 = \mathbf{99.74}\%$….(0.5p)…….

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Exercise 03 (07 p): Part 01 - Answer with True or False, and justify briefly each response, whether True or False **(03 p)**:

1- The numbers from 0.0003 to 0.99997 in the Z-table represent the PDF. **False.**(0.5p) The Z-table represents the cumulative probability (area under the curve) for the standard normal distribution, not the probability density function (PDF). The PDF is the derivative of the cumulative distribution function (CDF)……0.5p…………………………………..

2- The parameter determines the shape of the distribution. **True**. (0.5p) The parameters of a probability distribution determine its shape because they control key aspects of how the distribution behaves and how data is spread around the central value……(0.5p)……………………

3- A discrete random variable is a variable that can take countable values within a finite or infinite set….**True**.. (0.5p) because it represents outcomes that are distinct and separable……(0.5p)……………………………………………………………………………………

Part 02 - Answer questions briefly **(04 p)**:

1- Why do probabilities in the normal and standard normal distributions never reach exactly 1? Because as we move further away from the mean the probability value approaches zero and becomes negligeable: the tail of the normal distribution extend infinitely in both directions to get a probability closer and closer to 1, but never actually reaches it. (1p)

2- What is the connection between the negative binomial and binomial distribution? the binomial focuses on the number of successes within a fixed number of trials, while the negative binomial focuses on the number of trials required to reach a fixed number of successes……(1p)

3- Why does the binomial distribution use combinations without repetition despite relying on replacement ? because X doesn't repeat in the same trial. Combinations without repetition are used to calculate the probability accurately in fixed p but not in q…(1p)……………

4- Why does the binomial distribution use combinations instead of permutations in its PMF? because in the binomial distribution, we are interested in the possibilities (number of successes) regardless of the order……(1p)…………………………………