Abbas Lagrour University - Khenchela

Faculty of Economic, Commercial and Management Sciences

2nd Semester of the 2023/2024 university year first year students

An exam in statistics 2**(section A&B)**

Exercise 1 :

Suppose three coins are tossed. Let X be the random variable representing the number of tails that occur.

Required:

1. Find the sample space S ?
2. Find the cardinality of S?
3. Identify the set of possible values for the random variable X ?
4. Find the probabilities corresponding to the values of the random variable X?
5. Find the probability mass function of X ? and represent it graphically?
6. Find the cumulative distribution function? and represent it graphically?

Exercise 2 :

Consider the following cumulative distribution function of a random variable X:(**The first derivative of the CDF yields the PDF, F ′ (x) = f (x)**)

$$F(x) = \begin{cases} 0 \; ; \; if \; x < 2 \\ \frac{1}{4}x^2 + 2x - 3 \; ; if \; 2 \leq x \leq 4 \\ 1 \; ; \; x > 4 \end{cases}$$

Required:

1. What is the PDF of X?

2. Determine E(X) and Var(X).

1 - The number of ways in which they can be seated so that two men refuse to sit together is:

- The number of ways in which 3 men and 3 women can be seated at a table with 6 chairs is:

$$C_7^3 . C_8^3 = 35. 56 = 1960$$

- We consider these two men sitting together, so the number of approaches becomes:

$$C_8^3 . C_5^1 = 280 = 5. \ 56$$

- So the number of ways these two men don't come together at this table is:

$$1960-280= 1680 C_7^3 . C_8^3 - C_5^1 . C_8^3 =$$

2 - The number of ways in which they can be seated so that two women refuse to sit together is:

- The number of ways in which these two women meet is: $C_7^3 . C^1_{6= 35.6=210}$

- So the number of ways these two women don't come together at this table is:

$$C_7^3 . C_8^3 - C_7^3 . C_6^1 = 1960 - 210 = 1750$$

3 - The number of ways in which they can be seated so that a man and a woman refuse to sit together is:

- The number of ways in which they can be seated so that a man and a woman meet together is:

$$C^2_{7= 315} . C^2_6$$

- So the number of ways in which a man and a woman do not get together is:

$$C_7^3 . C_8^3 - C_6^2 . C_7^2 = 1960 - 315 = 1645$$

1.

$S = \{FFF, FFP, FPF, PFF, PPP, PPF, PFP, FPP\}$

2.

$|S| = 8.$

3.

4.

$P(x=0) = \frac{1}{8}$

$= \{0, 1, 2, 3\}$

$P(x=1) = \frac{3}{8}$

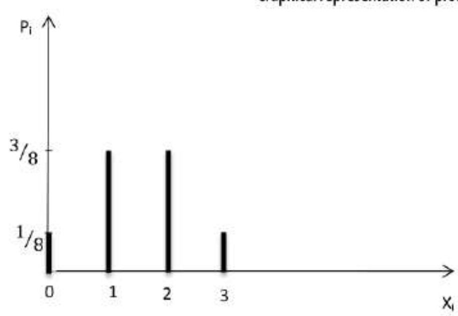
$P(x=2) = \frac{3}{8}$

$P(x=3) = \frac{1}{8}$

5.

x_i	0	1	2	3	$\sum$
P_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

Graphical representation of probability distribution:



6.

$\forall\ x \in]-\infty, 0[\Rightarrow F(x) = 0$

$\forall\ x \in [0, 1[\Rightarrow F(x) = \frac{1}{8}$

$\forall\ x \in [1, 2[\Rightarrow F(x) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$\forall\ x \in [2, 3[\Rightarrow F(x) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

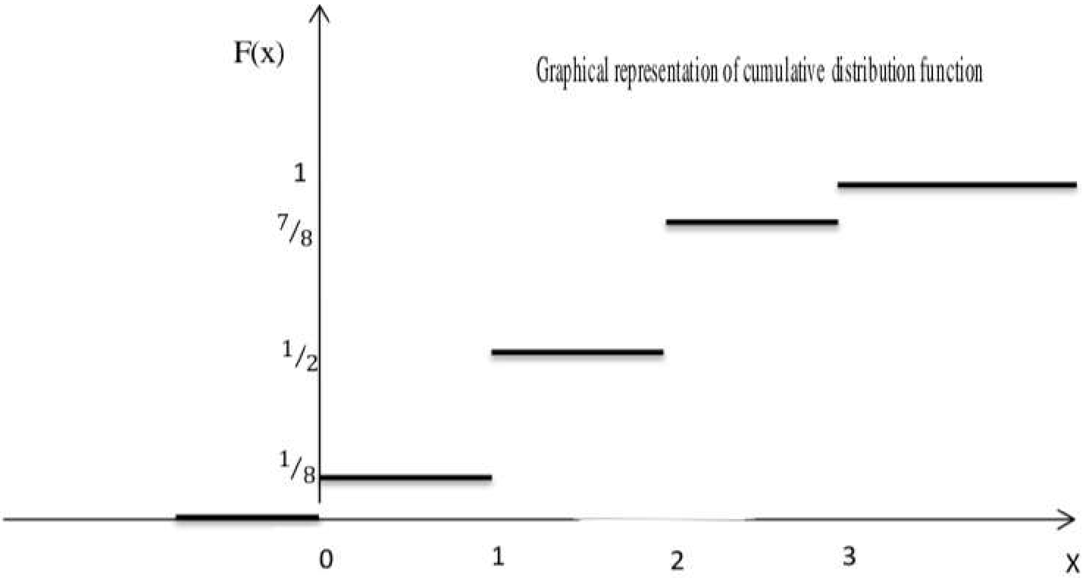
$\forall\ x \in [3, +\infty[\Rightarrow F(x) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

7.

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{7}{8}$$


F(x)
Graphical representation of cumulative distribution function
1
7/8
1/2
1/8
0
1
2
3
X

The first derivative of the CDF yields the PDF, $F'(x) = f(x)$:

$$f(x) = \begin{cases} 0 & \text{if } x < 2 \\ -\frac{1}{2}x + 2 & \text{if } 2 \leq x \leq 4 \\ 0 & \text{if } x > 4. \end{cases}$$

$$2\_\ \mathrm{E}(X) = \int_{-\infty}^{2} x\ f(x)\ \mathrm{d}x = \int_{-\infty}^{2} x0\ \mathrm{d}x + \int_{2}^{4} x\ \left(+\frac{1}{2}x + 2\right)\ \mathrm{d}x + \int_{4}^{\infty} x0\ \mathrm{d}x$$

$$= 0 + \int_{2}^{4} \left(+\frac{1}{2}x^2 + 2x\right) \mathrm{d}x + 0$$

Given that we have already calculated $\mathrm{E}(X)$, we can use Theorem 2 to calculate the variance as $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$. The expectation of X^2 is

$$\mathrm{E}(X^2) = \int_2^4 x^2 \left(+ \frac{1}{2}x + 2 \right) \mathrm{d}x = \int_2^4 \left(+ \frac{1}{2}x^3 + 2x^2 \right) \mathrm{d}x$$