Univ: Abbes Laghrour-Khenchela / Fac: ST / Dept: M / L3 Maths

Final Exam : Differential Geometry 21 Mai 2024 / H. RAMOUL

Exercice n=1 — 4 points

1. Show that a subset $M \subset \mathbb{R}^n$ is a 0-dimensional submanifold if and only if M is discrete, i.e. for every $p \in M$, there is an open set $U \subset \mathbb{R}^n$ such that $U \cap M = \{p\}$.
2. Show that a subset $M \subset \mathbb{R}^d$ is a d-dimensional submanifold if and only if M is open.

Exercice n=2 — 6 points

Let us consider the following set

$$M = \left\{(x, y, z) \in \mathbb{R}^3; \quad x^2 + 4y^2 = 1,\ z = x^2 - 4y^2\right\}.$$

1). Show that M is a submanifold of $\mathbb{R}^3$ and determine its dimension.
2). Find the tangent space of M at any point.
3). Give a basis for this tangent space.

Exercice n=3 — 10 points

Let $M_i \subset \mathbb{R}^{n_i}$ be two d_i-submanifolds of class $\mathcal{C}^{k_i}$ for $i = 1, 2$.
1. Show that $M_1 \times M_2$ is a $(d_1 + d_2)$-dimensional submanifold of $\mathbb{R}^{n_1+n_2}$ of class $\mathcal{C}^{\min(k_1,k_2)}$.
2. Let $a \in M_1$ and $b \in M_2$. Show that

$$T_{(a,b)}(M_1 \times M_2) = T_a M_1 \times T_b M_2.$$